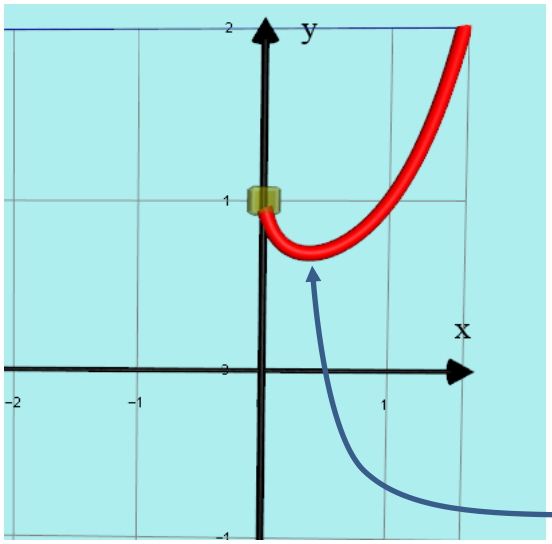
The graph of y = x^x

I have been working on this graph and I found the graph to be ABSOLUTELY fascinating.

For x > 0 there is no problem.

It looks like this:



If $y = x^x$
then $\ln(y) = \ln(x^x)$
so that $\ln(y) = x\ln(x)$
therefore differentiating:

$$\text{we get } \frac{1}{y}\frac{dy}{dx} = x \times \frac{1}{x} + 1 \times \ln(x)$$

$$= 1 + \ln(x)$$

If the gradient is zero then $\ln(x) = -1$
so $x = e^{-1} \approx 0.367879$ and $y = 0.6922$
This is a minimum point.

Many people believe that the minimum value of x^x is 0.6922…

but there is a left hand side to the graph when x< 0 where it gets very exciting!

If x < 0 we CAN find y values!

For instance if:

x = – 1 we get (– 1)^(– 1) = – 1

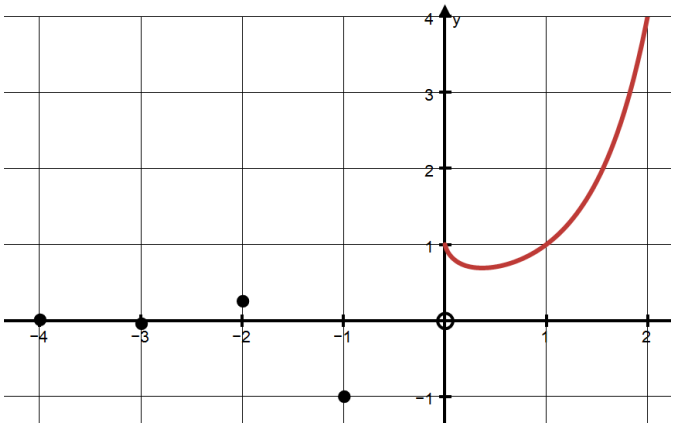
x = – 2 we get (– 2)^(– 2) = + ¼

x = – 3 we get (– 3)^(– 3) = – 0.037

x = – 4 we get (– 4)^(– 4) = + 0.0039

These are all REAL numbers and they are less than 0.6922!

If we plot these extra points we get something like this…



…and these are not just a few isolated points! I will explain below…

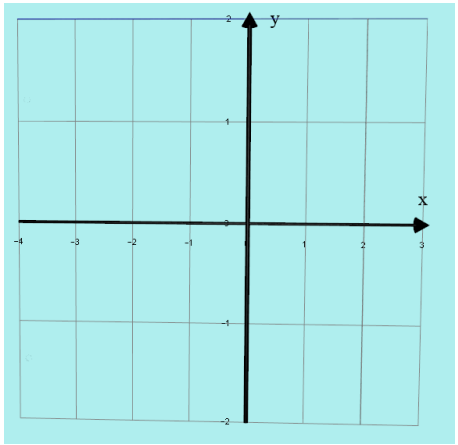
If x = –0.5 then y = (–0.5)^(–0.5) = ***– 1.414i***

If x = –1.5 then y = (–1.5)^(–1.5) = ***+0.544i***

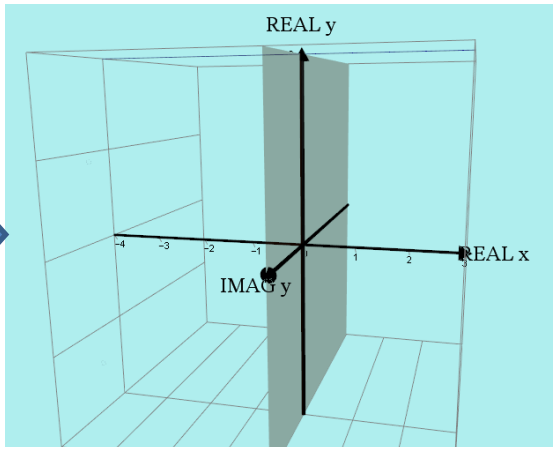
If x = –1.6 then y = (–1.6)^(–1.6) = ***0.15 + 0.45i***

We can plot these points if we change from the normal x, y axes to a REAL x axis sticking through a complex y plane as below:

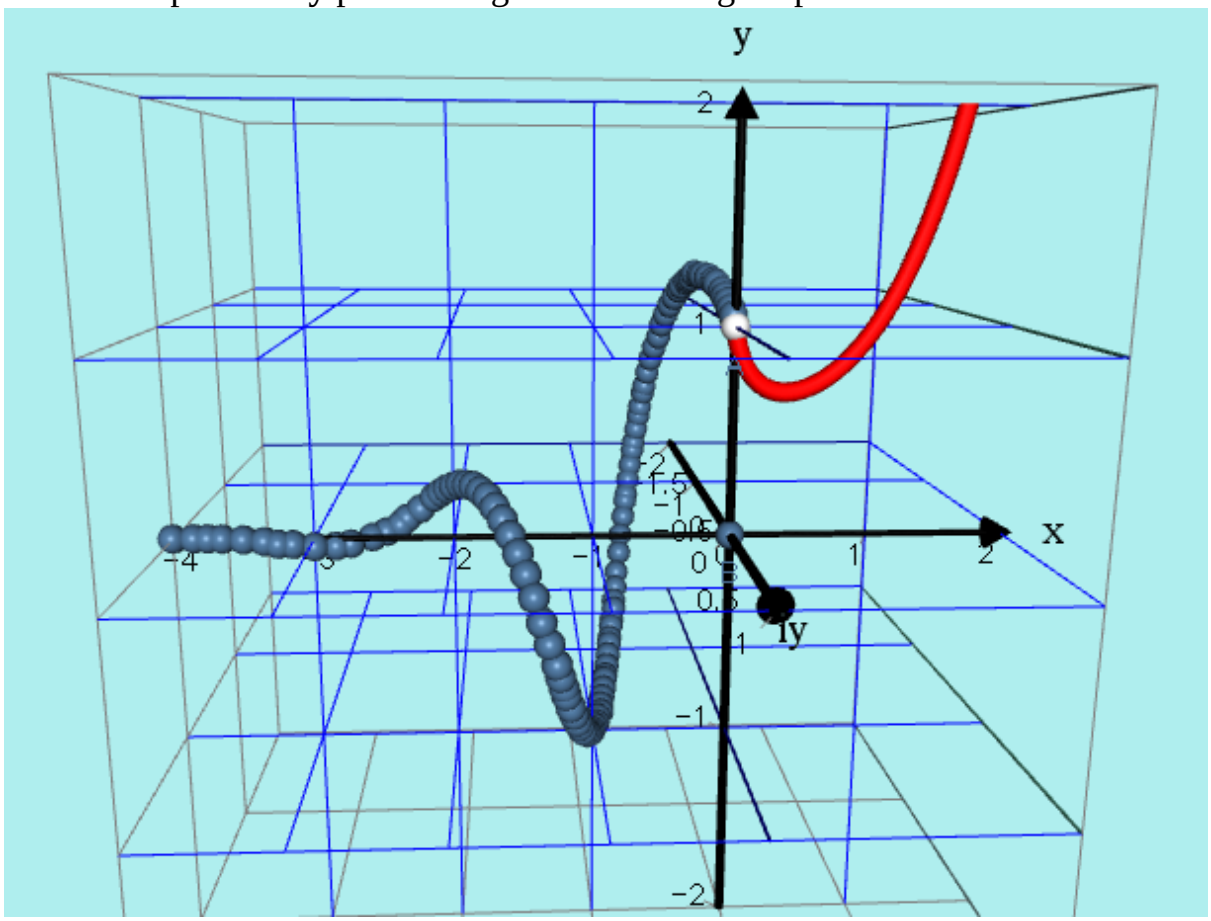
x, y axes



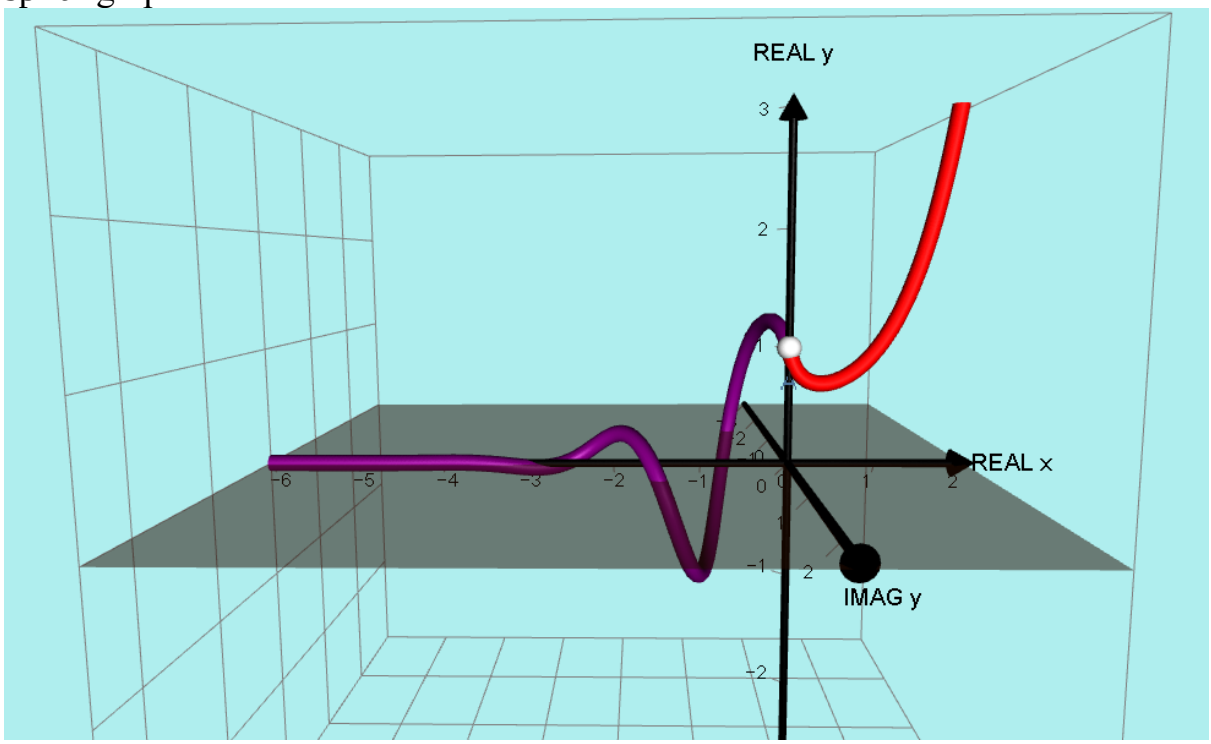
Real x axis and **complex y PLANE**



Now if we plot **many** points we get this amazing shape!



I finally found the equation of the actual spiral and produced this wonderful spiral graph…



Do watch this short video I made about this…

$y = x^x$ 2020

https://www.screencast.com/t/Vkz2HWA27